Inference on cointegrating rank and (nonlinear) cointegrating relations in nonlinear SVARs: some preliminary results

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Abstract. Building on recent extensions of the Granger–Johansen representation theorem to nonlinear structural VARs (Duffy et al., 2023; Duffy and Mavroeidis, 2024), this paper considers asymptotic inference on the cointegrating rank and (possibly nonlinear) cointegrating relationships in a first-order CKSVAR: i.e. a two-regime, piecewise linear structural VAR, of the form

$$\phi_0^+ y_t^+ + \phi_0^- y_t^- + \Phi_0^x x_t = c + \phi_1^+ y_{t-1}^+ + \phi_1^- y_{t-1}^- + \Phi_1^x x_{t-1} + u_t, \qquad u_t \sim_{\text{i.i.d.}} N[0, \Sigma],$$

where $y_t^+ := \max\{y_t, b\}$ and $y_t^- := \min\{y_t, b\}$ for some (known) threshold b, and $x_t \in \mathbb{R}^{p-1}$. In the special case where $\phi_0^+ = \phi_0^-$, the model parameters may be estimated by OLS, and the Gaussianity assumption on $\{u_t\}$ may be dispensed with; but otherwise the presence of a nontrivial nonlinear mapping on the l.h.s. of the model precludes OLS, in which case we consider estimation by maximum likelihood (under Gaussianity). We show that likelihood-based estimators of the cointegrating relationships enjoy an elevated rate of convergence, and are asymptotically mixed Gaussian when the correct cointegrating rank is imposed on the model, thereby generalising classic results from the linear cointegrated VAR to present setting. On the computational side, we demonstrate how reduced rank regression can be used to facilitate maximisation of the likelihood under a given cointegrating rank, even in cases where the nonlinearity of the l.h.s. precludes the direct application of that method.

Duffy, J. A. and S. Mavroeidis (2024): "Common trends and long-run identification in nonlinear structural VARs," arXiv:2404.05349.

Duffy, J. A., S. Mavroeidis, and S. Wycherley (2023): "Cointegration with Occasionally Binding Constraints," arXiv:2211.09604v2.